Advanced Math: Notes on Lessons 45-49
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Lesson 45: Conditional Permutations / Linear Regression

Remember that a permutation is an arrangements in a specific order, where you can't repeat the same item. Most problems with permutations involve reporting the number of permutations.

If there are no special conditions on a permutation, you can use the equation you learned earlier; the number of permutations (possible sequences) of a set of *n* distinct things, taken *r* at a time, is:

$$_nP_r = P(n,r) = \frac{n!}{(n-r)!}$$

But if there are special conditions on the permutations, it's not so simple. A good way to figure out the number of possible alternatives is to draw boxes (one for each position), and figure out the number of different possible values in each position. Start with the most limited values, and then go from there. Once you find them all, multiply them together.

Sometimes even that won't work. The obvious example is when you have the case of 2 boxes (positions), 3 boxes, 4 boxes, and so on . In those cases, you need to divide the possibilities into these different cases (2 boxes, 3 boxes, etc.), figure out each one in turn, and then add them all together.

E.G., "Given only digits 1-5, how many positive odd numbers are there less than 10,000 with no repeating digits?"

"Less than 10,000" includes 1,2,3, and 4-digit numbers, so we have a varying number of boxes.
1 box: We can "eyeball" this. Only 1,3, and 5 are permitted (it's odd), so total possibilities = 3.
2 boxes: With two boxes, the lower digit must be 1,3, or 5 (it's odd), so there are only 3 possible values. The upper digit can be any of 1 through 5, but it can't be the same as the lowest, so that's 5-1=4 possible values. Total possibilities = 4 * 3 = 12.
3 boxes: Lowest has only 3 values. Second lowest can be any of 5-1=4, and the highest can be any of 4-1=3. Total = 3*4*3 = 36.
4 boxes: Lowest has 3 values, second lowest 4, next lowest 3, next lowest 2. Possibilities = 2*3*4*3 = 72.

Total possibilities = 3 + 12 + 36 + 72 = 123.

Lesson 46: Complex Roots/Factoring over the Complex Numbers

Real numbers are a subset of the complex numbers (a real number is just a complex number with 0i as the imaginary part). Most of the operations you've been doing with real numbers also work with complex numbers.

For example, if you were told that a quadratic equation had roots 2 and -3, you could figure out the original equation if the x^2 coefficient is 1 by doing this:

$(x - 2)(x - (-3)) = (x-2)(x+3) = x^2 + x - 6$

The same thing works for complex numbers, but *watch the sign* - you're *subtracting* the number, so make sure that *both* the real and imaginary number coefficients have their signs reversed. E.G.:

If 2+3i and 2-3i are the roots of a quadratic equation with x^2 coefficient of 1, what's the equation?
$(x - (2 + 3i))(x - (2 - 3i)) = (x - 2 - 3i)(x - 2 + 3i) = x^2-4x-9i^2+4 = x^2-4x+9+4 = x^2-4x+13$

You can factor arbitrary quadratic polynomials, too. The trick is to first factor out the x^2 coefficient (and don't forget it - you'll need it later). Then use the quadratic equation to find the other roots, and write them down. So to factor $2x^2-8x+26$...

First divide by two:
$= 2(x^2-4x+13)$
Then use the quadratic equation, which will tell you that its roots are 2+3i and 2-3i. Write that out:
$2(x-(2+3i))(x-(2-3i)) = 2(x-2-3i)(x-2+3i)$

Lesson 47: Vertical Sinusoid Translations / Arctan
You can move graphs up and down easily. Replace y with y-k in an equation will move it up by k units; replace y with y+k will move it down k units. Usually you will then need to solve for y, e.g.:

Original: $y = x^2$

Move it up by 3 units, by replacing y with y-3:
$y-3 = x^2$
$y = x^2+3$

Most of the time your equation will have only one y, and it's the one on the left-hand-side. Moving the "k" to the other side with change the sign, but give you an added or subtracted number that moves the equation up or down:
y = stuff
Move up by k units:
y-k = stuff
y = stuff + k

Recognize this format - when you have some equation in y = stuff + k form, it's just stuff moved up by k units.

The Arctan portion just reminds us about the big limitation of Arctan - it can only produce values inside -90°..90°.

Lesson 48: Powers of Trig Functions / Perpendicular Bisectors
This just introduces a common trig notation. We often need to use a trig function and then square the result; a shorthand for this is putting the exponent right after the name of the trig function. Thus:
$(\sin x)^2 = \sin^2 x$

You learned how to find a perpendicular bisector earlier (lesson 37). Another way to do it, which Saxon calls the "midpoint formula" method, is to find the midpoint, find the new slope (-1/oldslope), and then use y=mx+b with the midpoint's x and y to find b.

Lesson 49: The logarithmic function / Development of the rules for logarithms

What does the logarithmic function actually look like?

Well, if
$y = \log_b x$
that's just another way of saying (applying b^{equation} to both sides):
$b^y = x$

"$\log_b N$" just returns "how many times you have to multiply *b* by itself to compute N".

For base 2, this should be obvious for a few cases:
$\log_2 1 = 0$ because $2^0 = 1$
$\log_2 2 = 1$ because $2^1 = 2$
$\log_2 4 = 2$ because $2^2 = 4$
$\log_2 8 = 3$ because $2^3 = 8$

You can then draw it out. All logarithms have the same characteristic shape.

The second part of this lesson tries to justify the logarithm rules we memorized earlier (lesson 40).

$$\log_b(x \cdot y) = \log_b x + \log_b y$$
$$\log_b \frac{x}{y} = \log_b x - \log_b y$$
$$\log_b x^y = y \log_b x$$