<u>Advanced Math: Notes on Lessons 17-20</u>
<u>David A. Wheeler, 2007-09-19</u>

This class is designed to prepare your for additional mathematics, should you choose to take/learn them. Two areas in particular that this course prepares you for are:

- **Calculus.** This is the study of derivatives (rate of change) and integrals (areas & volumes). That is, calculus studies continuously-changing values. Another course called "differential equations" builds further on top of calculus, enabling the study of more complex systems that continuously change. Calculus turns out to be marvelous at modeling the physics of the real world,. including astronomical motion, electronics, and so on, and it's useful for modeling other things too. Isaac Newton and Gottfried Leibniz independently developed calculus, and its development was so gigantically important that there was a *huge* controversy over who invented it first (with national pride at stake: Newton was English, while Leibniz was German). The controversy was so bitter that it divided mathematicians for many years. Leibniz spent a great deal of time working out exactly how to notate calculus; today most people use Leibniz' clever notation.
- **Discrete mathematics** (including logic). Discrete mathematics involves the study of objects and ideas that can be divided into separated or discontinuous parts, including logic/logic puzzles. Digital computers are based on discrete mathematics (1 and 0 are discrete from each other, and they form the basis of digital computing). You should learn discrete mathematics if you're going to develop computer software in a serious way.

Lesson 17: Pythagorean Theorem / Similarity
17A is about proving the Pythagorean theorem.

"Similar" (marked as ~) means "same shape, not necessarily same size". Similar shapes can be *scaled* to be congruent with another. Congruence is actually a special case of similarity (e.g., the scaling factor is 1); two similar triangles may also be congruent. There are several postulates that can prove similarity, but beware, though they *look* like the congruence postulates (they *are* related), they aren't the same postulates. So, to prove that two triangles are similar, you need to show SSS (all three sides *similar*), SAS (sides are *similar* and angle is congruent), or AAA (all three angles congruent). AAA is listed separately from SSS and SAS from the book, oddly enough. When you're giving reasons in a proof, use names like "SSS similarity postulate" to distinguish it from the "SSS congruence postulate".

Note that ~ means similar, = means equal to, and ≅ means "congruent". The traditional symbol for congruent is no accident; it's supposed to help you remember that congruent means "similar shape with equal scale".

Lesson 18: Advanced Word Problems
Word problems are *really* important. "Word problems" are simply exercises in applying math; if you can't do word problems, you can't apply the math to the real world, and you probably don't understand the math either. You *must* have as many variables in your equations as you have independent values, and if you have a system of linear equations (all with exponents of 1), you have to have as many equations as you have variables to get a solution.

Walk though each example carefully, and make sure you can re-do each.

Lesson 19: Nonlinear Systems/ Factoring Exponentials/ Sum and Difference of Two Cubes
Linear equations only have variables with an exponent of 1; nonlinear equations are "everything else". Quadratic equations are nonlinear; you'll probably need the quadratic equation to solve them.

Factoring exponentials – this can get a little mind-bending; do this in slow steps or you'll probably make a mistake. Remember that $x^{a+b} = x^a x^b$.

$a^3+b^3 = (a+b)(a^2 - ab + b^2)$ and $a^3-b^3 = (a-b)(a^2 + ab + b^2)$
Since these are hard to remember, Saxon shows a trick for rederiving them quickly... which should also prove to you that they're true! [Walkthrough: Show how to compute a^3+b^3 / (a+b)... the trick is to write the $+b^3$ term waaaay later].

Lesson 20: Two Special Triangles (45-45-90 and 30-60-90)
This lesson shows two special triangles (the 45-45-90 and 30-60-90) triangles, and derives their sin, cos, and tangent values. The key is that you should be able to redo these derivations (e.g., figure out tan 30° by creating a 30-60-90 triangle); if you can, you'll start to really understand the trig functions (and that's important). These are important "reference triangles" - you can buy triangles of with these angles for manual drafting, etc. The basic idea is straightforward, in both cases based on the isosceles triangle (2 angles equal).

[Walkthrough] Obviously the 45-45-90 triangle has two equal angles, so it's an isosceles triangle. You can then use Soa Cah Toa to find the trig values.

[Walkthrough] You can take an equilateral triangle, bisect one of its angles, and thus create two 30-60-90 triangles. With that, you can figure out the trig values for a 30-60-90 triangle.

Saxon prefers writing $\frac{\sqrt{3}}{3}$ instead of $\frac{1}{\sqrt{3}}$, i.e., he prefers the denominator to be simple, even if it makes the numerator more complicated. Saxon complains that "Some modern authors have suggested that rationalizing the denominator is an unnecessary procedure. Perhaps it is...". Let me make it clear that I'm one of those "modern authors", and in fact I don't know anyone other than Saxon who insists on this. I don't see any advantage to rationalizing the denominator. So, I will accept either form as a simplified form (with full credit). However, if you want the answer to look just like Saxon's, you'll need rationalize the denominator, and that might make your parents' job easier. It's easy to do this, simply multiply by the denominator/denominator (=1).

THIS WEEK: Work through lessons 17-20 this week ("Monday – Thursday").

Do home study test #4 this week (covers lessons 13-16), presumably on Friday. It will use some trig functions, so you *will* need a calculator that has buttons for sin, cos, and tan on it. Any scientific calculator (even the $10 kind) will have these functions. Please review your notes before taking it, and memorize what you need first. *Show your work*; I can't give partial credit without that!

Have a parent grade it initially (to identify which ones are right or wrong), and bring it in to the next class. I intend to do the final grading, so that I can give partial credit (or full credit if it's just an equivalent expression).