Advanced Math: Notes on Lessons 110-113

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Lesson 110: Graphs of arcsine and arccosine / Graphs of arcsecant and arccosecant / Graphs of arctangent and arccotangent

This is all about learning what the graphs of the arc-trig functions look like. I don't have anything to add to this; examine the graphs in the book.

Lesson 111: Logarithmic Inequalities

If you have an inequality, and a logarithm with an unknown as its parameter, you need to do an antilogarithm (exponentiate). But there are two catches that you have to account for:

1. Logarithms only accept positive values (>0).
2. Exponentiation using a value less than 1 reverses the inequality (just like multiplication or division by a negative number does).

This is easier to understand through examples. For example, let's try to solve:

$$\log_5(x-3)<6$$

We can exponentiate the whole expression, exponentiating it by 5 to reverse the logarithm. Since 5 is more than 1, we won't have the "value less than 1" problem, leading us this way:

$$\log_5(x-3)<6$$

$$5^{\log_5(x-3)}<5^6$$

$$x-3<15625$$

$$x<15628$$

But we are *not* done, because logarithms only accept positive values. The original expression has a logarithm accepting "x-3", and we can only accept x-3 if it is greater than 0. So let's solve for this requirement:

$$x-3>0$$
$$x>3$$

For the final answer, *both* must be true – we have to have the solution, and it has to be legal in the original expression. So the final answer is:

$$x<15628 \text{ and } x>3$$
$$x<15628 \text{ and } 3<x$$
$$3<x<15628$$

Let's try another, one that will use both rules:

$$\log_{1/3}(x-7)<2$$

$$(1/3)^{\log_{1/3}(x-7)}>(1/3)^2$$

$$x-7>\frac{1}{9}$$

$$x>7\frac{1}{9}$$

Notice that the exponentiation above reversed the inequality, because we raised the expression by a value less than 1. Again, we need to account for logarithm, which requires values to be positive. In this case, the expression accepted by the logarithm is "x-7", so we need to determine when this is positive. "x-7>0" easily solves to "x>7". Now we combine them:

$x>7\frac{1}{9}$ and $x>7$

$$x>7\frac{1}{9}$$

Lesson 112: Binomial Theorem

The "binomial theorem" provides a quick way for finding the terms of $(a+b)^n$ for large values of n. It looks *way* more complicated than it is.

First, here's a formal definition, similar to the one given in the book: If n is a positive integer, and k is a positive integer less than or equal to n+1, the k-th term of $(F+S)^n$ is:

$$\frac{n!}{(n-k+1)!(k-1)!}F^{n-(k-1)}S^{k-1}$$

In practice, this is easier to use right-to-left:

1. First, find the exponent of S, which is just k-1 (one less than the position you're finding).
2. Second, find the exponent of F. The sum of F and S's exponents must always be n, so F's exponent is really just n-(S's exponent). The book's formal definition is slightly different, in that this isn't as obvious the way Saxon writes it.
3. Finally, find the coefficient. The numerator is just n!; the two expressions in the denominator are simply factorials of the values of the exponents of F and S.

So, to find the 11^{th} term of $(F+S)^{15}$:

1. First, find the exponent of S, which is just k-1 = 11-1 = 10
2. Second, find the exponent of F = n-(S's exponent) = 15-10 = 5
3. Finally, find the coefficient:

$$\frac{15!}{5!\,10!}=\frac{11\cdot 12\cdot 13\cdot 14\cdot 15}{5!}=\frac{360360}{120}=3003$$

So the answer is $3030F^5S^{10}$.

Lesson 113: Synthetic Division / Zeros and Roots

Synthetic division is a fast way of dividing polynomials for a special case: dividing by *x-a* where *a* is constant and *x* is the variable of the polynomial.

For example, you should be able to do polynomial division of x^3-5x^2+5x+7 by x+4, producing:

$$x^2 - 9\text{x} + 41 - \frac{157}{x+4}$$

But that takes a while. In synthetic division, you first write down the constant *a* in a half-box, followed by the coefficients for the polynomial in the numerator. Since we divide by "x-a", if you have an expression like "x+a", you write down the negative of the value you're adding (in this case -4):

$$\underline{-4|}\quad 1 \quad -5 \quad 5 \quad 7$$

Then, to do synthetic division, copy the first coefficient two rows down. Then repeatedly multiply the bottom number by the divisor constant, record that result under the next coefficient, and add them up, repeatedly. E.G.:

$$\begin{array}{r|rrrr} \underline{-4} & 1 & -5 & 5 & 7 \\ & \downarrow & -4 & 36 & -164 \\ \hline & 1 & -9 & 41 & -157 \end{array}$$

In this example, we first copy the leftmost "1" to the bottom (third) row. We then multiply 1 by -4, and record -4. Add up -5 and -4, yielding -9. Now multiply -9 by -4, and record 36. Add 5 and 36, yielding 41. Finally, multiply 41 by -4, and record -164. Add 7 and -164, yielding -157.

The bottom row is the result of the division – the last value is the remainder. Remember that when you divide a polynomial by some value x-a, you're going to have a new degree that's one less than when you started, so the final result here (properly interpreted) is the same:

$$x^2 - 9\text{x} + 41 - \frac{157}{x+4}$$

The terms "a zero" and "a root" can be confusing, because they have very similar meanings:

- Use "a zero" for a *polynomial*; "a zero of a polynomial" is a value for *x* that produces 0 for the polynomial
- Use "a root" for an *equation* (has an "=" sign); "a root of an equation" means that the equation is in the form *stuff=0*, and the root of an equation is a value of the unknown *x* that solves the equation. If *stuff* is a polynomial, then it's a polynomial equation.

You can use synthetic division to determine if a given value *c* is a zero of a polynomial, or a root of a polynomial equation. Just use *c* inside the box (without negating it), and see if the remainder is zero – if it is, then *c* is a zero for the polynomial and a root for the (polynomial) equation. For example, given the polynomial x^3-12x^2+47x-60, you can show that 3 is a zero of this polynomial:

$$\begin{array}{r|rrrr} \underline{3} & 1 & -12 & 47 & -60 \\ & \downarrow & 3 & -27 & 60 \\ \hline & 1 & -9 & 20 & 0 \end{array}$$

This means that 3 is also a root for the equation x^3-12x^2+47x-60 = 0.